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A kinematic method for the computation of the natural modes of Fluid–Structure Interaction systems

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Abstract

A kinematic approach is elaborated for Fluid–Structure Interaction (FSI) in linear conservative systems. It is based on the concept of added fluid field, which stands for the fluid displacement explicitly or implicitly associated with the structural displacement in a purely mechanical approach. The Lagrangian of a coupled system is hence written as the sum of the Lagrangian of the structure, the acoustic Lagrangian and a coupling Lagrangian expressed as the mass and stiffness-weighted scalar products of the acoustic field and the added fluid field.

It is shown that the coupled modes of a fluid–structure system can be computed by combining its uncoupled modes, with a procedure involving symmetric matrices. Indicators of the coupling strength are provided: it is shown that strongly coupled systems do not exhibit large variations of their behavior when their uncoupled natural frequencies coincide, whereas weakly coupled systems do. Applications to some case studies of FSI are provided.

Keywords: Fluid–Structure Interaction; Natural modes

1. Introduction

Fluid–Structure Interaction (FSI) is present in many fields of pressure vessels and piping, at the design stage as well as in troubleshooting investigations. The identification of the natural modes of fluid–structure systems is a concern in the analysis of turbulence-induced vibrations (Weaver et al., 2000; Au-Yang, 2001), in the design of pipes against fluid transients (Tijsseling, 1996; Wiggert & Tijsseling, 2001; Koelle, 2004) and in vortex-shedding cases (Naudascher and Rockwell, 1994; Blevins, 2001).

Rigorously speaking, a numerical simulation for determining the natural modes of a coupled system should involve the mechanics and the acoustics at the same time. In dedicated computer codes, a variational formulation is generally used, which is based on the kinematics for the structure, and on the pressure for the fluid (Paidoussis, 1998; Fahy, 1985; Gibert, 1988). More complex formulations such as the "u, p, Φ" method have sometimes been introduced to improve the computational efficiency (Morand and Ohayon, 1992). These methods have been extensively validated and provide reference solutions. However, they require powerful computers even for simple structures, and the modelling itself must be made by skilled engineers. For this reason, many commercial codes do not incorporate FSI analysis.

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Nomenclature

A_p	acoustic displacement amplitude of the pth mode
$\mathscr{C}_k(\boldsymbol{\alpha}, \boldsymbol{\Phi}(\boldsymbol{\xi}))$	nondimensional stiffness-weighted spatial correlation of the displacement fields $\boldsymbol{\alpha}$ and $\boldsymbol{\Phi}(\boldsymbol{\xi})$
$\mathscr{C}_m(\boldsymbol{\alpha}, \boldsymbol{\Phi}(\boldsymbol{\xi}))$	nondimensional mass-weighted spatial correlation of the displacement fields $\boldsymbol{\alpha}$ and $\boldsymbol{\Phi}(\boldsymbol{\xi})$
c	speed of sound in the fluid (m/s)
c_s	speed of the compressive waves in the structure (m/s)
E	Young's modulus of the structure (MPa)
F_ω	amplitude of the excitation force, defined by $F_\omega = \mathbf{F}_\omega \cdot \mathbf{X}^f / X^f$
k	modal stiffness of the structure
k^a	acoustic stiffness, defined by $k^a A^2 = \mathbf{A}\mathbf{k}^f\mathbf{A}$
k^{add}	modal added stiffness, defined by $k^{\text{add}}X^2 = \boldsymbol{\Phi}(\mathbf{X})\mathbf{k}^f\boldsymbol{\Phi}(\mathbf{X})$
k^c	coupling stiffness, defined by $k^c AX = \mathscr{R}e[\boldsymbol{\Phi}(\mathbf{X})\mathbf{k}^f\mathbf{A}]$
k_n	stiffness of the nth structural mode
K	kinetic energy of the system
K^f	kinetic energy of the fluid subsystem
K^s	kinetic energy of the structure
m	modal mass of the structure
m^a	acoustic mass, defined by $m^a A^2 = \mathbf{A}\mathbf{m}^f\mathbf{A}$
m^{add}	added mass, defined by $m^{\text{add}}X^2 = \boldsymbol{\Phi}(\mathbf{X})\mathbf{m}^f\boldsymbol{\Phi}(\mathbf{X})$
m^c	coupling mass, defined by $m^c AX = \mathscr{R}e[\boldsymbol{\Phi}(\mathbf{X})\mathbf{k}^f\mathbf{A}]$
m_n	mass of the nth structural mode
$\mathbf{n}$	unit vector normal to a surface
p	pressure field
Q	Rayleigh quotient (Hz^2)
S_f	fluid cross-sectional area of a pipe (m^2)
S_s	cross-sectional area of a pipe structure (m^2)
U^{int}	interaction energy of the fluid and the structure
U	elastic energy of the system
U^f	elastic energy of the fluid subsystem
U^s	elastic energy of the structure
X_n	structure displacement amplitude of the nth mode
η^k	nondimensional stiffness coupling indicator, defined by $\eta^k = \mathscr{C}^k[\mathbf{A}, \boldsymbol{\Phi}(\mathbf{X})]\sqrt{k^{\text{add}}/(k + k^{\text{add}})}$
η^m	nondimensional mass coupling indicator, defined by $\eta^m = \mathscr{C}^m[\mathbf{A}, \boldsymbol{\Phi}(\mathbf{X})]\sqrt{m^{\text{add}}/(m + m^{\text{add}})}$
Θ	coupling indicator defined as the ratio of the interaction energy U^{int} of the fluid and the structure, and of the total energy of the system
κ	stiffness ratio equal to $k/(k + k^{\text{add}})$
μ	mass ratio equal to $m/(m + m^{\text{add}})$
ρ_f	fluid density (kg/m^3)
ρ_s	structure density (kg/m^3)
ω	circular frequency (Hz)
ω_a	Rayleigh frequency for the acoustic fluid, defined by $\omega_a^2 = k^a/m^a$ (ω_a is in Hz)
ω_e	natural frequency of a coupled mode (Hz)
ω_f	Rayleigh fluid frequency, defined by $\omega_f^2 = \omega^2 U^f(\mathbf{X}^f)/K^f(\mathbf{X}^f)$ (ω_f is in Hz)
ω_{red}	nondimensional frequency in a pipe calculation
ω_s	Rayleigh structure frequency, defined by $\omega_s^2 = \omega^2 U^s(\mathbf{X})/K^s(\mathbf{X})$ (ω_e is in Hz)
Ω	Rayleigh frequency of the structure with its added fluid, defined by $\Omega^2 = (k + k^{\text{add}})/(m + m^{\text{add}})$ (Ω in Hz)

Displacement fields, vectors and operators

$\mathbf{A}$	acoustic displacement field
$\mathbf{F}_\omega$	excitation force of the linear system
$\mathbf{k}^f$	stiffness operator of the fluid
$\mathbf{m}^f$	mass operator of the fluid

$\mathbf{n}_{fs}$	unit vector normal to the fluid/structure interface, directed from the fluid towards the structure
$\mathbf{n}_{sf}$	unit vector normal to the fluid/structure interface, directed from the structure towards the fluid
$\mathbf{X}$	structure displacement field
$\mathbf{X}^f$	fluid displacement field
$\boldsymbol{\alpha}_p$	pth nondimensional acoustic mode
$\boldsymbol{\xi}_n$	nth nondimensional structure with added fluid mode
$\boldsymbol{\Phi}(\mathbf{X})$	added fluid field associated with the structure field displacement $\mathbf{X}$

The study deals with the determination of the natural modes of a fluid–structure system when only uncoupled calculations can be performed. A related issue is the determination of the features of systems which would expose them especially to FSI (Leslie and Vardy, 2001, 2003); the only widespread rule stipulates that the coincidence of structural and acoustic natural frequencies must be avoided (API 618, 1995), but the physics of the coupling phenomena associated with this recommendation are not explicit.

Within the framework of linear acoustics and mechanics, a kinematic approach for fluid–structure systems is proposed, i.e., a formulation where the variables are displacements of the fluid and the structure, where the energy is expressed as a function of the displacements only, and where the pressure is not used. An expression of the energy as a sum of kinetic terms $\frac{1}{2}m_{ij}\omega^2 X_i X_j$ and elastic terms $\frac{1}{2}k_{ij}X_iX_j$ is then required, where the coordinates X_i are independent, and where m_{ij} is a mass matrix and k_{ij} a stiffness matrix. The definition of independent coordinates X_i is not straightforward, because the boundary conditions at the fluid–structure interface demand the continuity of the normal displacements. The key idea is to define a set of independent coordinates with the help of the ‘‘added fluid’’, which stands for the implicit or explicit fluid displacement associated with a structural displacement.

Section 2 is the exposition of the method: it introduces the added fluid concept and it provides an expression of the coupled modes of a fluid–structure system deduced from its uncoupled modes, based on a symmetrical matrix formulation. A dynamic balance between the amplitudes of the acoustic field and the structure field of a coupled mode is proposed in Section 3. In Section 4, the physics of the coupling phenomena are discussed: the physical coupling is defined as the ratio of the interaction energy and the total energy, and this coupling is compared to the spatial correlation of the uncoupled modes. It is shown that a weakly coupled system excited by fluid forces (e.g., turbulence-induced vibrations) is prone to strong variations of its vibrations if its uncoupled natural frequencies come to coincide, whereas a strongly coupled system is less sensitive to the value of its uncoupled natural frequencies. Section 5 deals with applications of the kinematic approach to some well-known cases of FSI, namely coaxial cylinders with interstitial fluid, a straight pipe with closed ends, low-frequency coupling in a Z-shaped pipe, and the dispersion relation in straight pipes submitted to shell deformations and nonplanar acoustic propagation.

2. Kinematic formulation for fluid–structure interactions

The starting point of the study is the determination of the mass and of the stiffness matrix of a coupled system. Let the fluid–structure relations at the interface be first considered for that purpose, denoting by $\mathbf{X}^f$ the fluid displacement field, and by $\mathbf{X}$ the structural displacement field. In the framework of inviscid fluids and linear mechanics, several books (Gibert, 1988; Axisa, 2001) suggest the use of the continuity of displacements at the interface written as

$$\mathbf{X}^f \cdot \mathbf{n} = \mathbf{X} \cdot \mathbf{n}, \tag{1}$$

and the balance of mechanical and fluid pressure at the interface

$$p\mathbf{n}_{fs} = \boldsymbol{\sigma} \cdot \mathbf{n}_{sf}.$$

From an energy point of view, the two above relations are equivalent. This can be shown by considering a small displacement of the interface δX in the fluid-to-structure direction: if the fluid and the structure stick together (continuity of displacement), the total work per unit area is the sum of the fluid pressure $p\delta X$ and the solid normal stress $-\sigma\delta X$. The principle of virtual work implies the balance of these two pressures. As a consequence, there is no need to mention the pressure balance at the interface in a kinematic approach, and the kinetic and elastic energies of the coupled system are the mere sums of the fluid and of the structure energies,

$$K = K^f(\mathbf{X}^f) + K^s(\mathbf{X}) \quad \text{and} \quad U = U^f(\mathbf{X}^f) + U^s(\mathbf{X}),$$

where K^f and U^f are respectively the kinetic and elastic energies of the fluid, and K^s and U^s are respectively the kinetic and elastic energies of the structure. Furthermore, the fluid energies are expressed as quadratic functions of the displacement fields in harmonic regime, the displacement fields being now complex vectors

$$K^f = \frac{\omega^2}{2}\mathbf{X}^f\mathbf{m}^f\mathbf{X}^f \quad \text{and} \quad U^f = \frac{1}{2}\mathbf{X}^f\mathbf{k}^f\mathbf{X}^f,$$

where the mass operator $\mathbf{m}^f$ and the stiffness operator $\mathbf{k}^f$ are shorthand notations for volume integrals, defined for a couple of fluid fields $\mathbf{X}_1^f$ and $\mathbf{X}_2^f$ by

$$\mathbf{X}_1^f\mathbf{m}^f\mathbf{X}_2^f = \int \rho_f \mathbf{X}_1^{f*}\mathbf{X}_2^f \,\mathrm{d}v \quad \text{and} \quad \mathbf{X}_1^f\mathbf{k}^f\mathbf{X}_2^f = \int \rho_f c^2 \mathrm{div}(\mathbf{X}_1^f)^* \mathrm{div}(\mathbf{X}_2^f)\,\mathrm{d}v,$$

and where ρ_f is the fluid density, c is the fluid speed of sound, and the complex conjugation is used in the energy expression as classically required by harmonic analysis.

The continuity equation (1) makes the fields $\mathbf{X}^f$ and $\mathbf{X}$ dependent, so that another set of variables is required to determine the mass and stiffness matrix of the coupled system. Let the fluid field be expressed for that purpose as the sum of an acoustic field $\mathbf{A}$, i.e., a displacement field which vanishes at the fluid–structure interface, and of an "added fluid" field with a normal component equal to the structural displacement at the interface, and arbitrarily chosen elsewhere. This added field $\mathbf{\Phi}(\mathbf{X})$ depends only on the structure displacement, and one gets

$$\mathbf{X}_f = \mathbf{\Phi}(\mathbf{X}) + \mathbf{A}. \tag{2}$$

This expansion is illustrated in Fig. 1, in the case of a cylinder filled with quiescent liquid submitted to nonplanar acoustic waves: the first drawing is a structural mode with its associated fluid field, the second the fluid mode with no displacement at the outer boundary, and the third mode is the superposition of the two former ones (the structural mode is drawn with five lobes and the fluid mode with seven lobes purely for purposes of illustration).

Expressions of the energies suited to the determination of the mass and of the stiffness matrices can now be carried out, with the help of the fluid field expansion

$$K = \frac{\omega^2}{2}\mathbf{A}\mathbf{m}^f\mathbf{A} + K^s(\mathbf{X}) + \frac{\omega^2}{2}\mathbf{\Phi}(\mathbf{X})\mathbf{m}^f\mathbf{\Phi}(\mathbf{X}) + \omega^2\mathcal{Re}[\mathbf{\Phi}(\mathbf{X})\mathbf{m}^f\mathbf{A}], \tag{3}$$

$$U = \frac{1}{2}\mathbf{A}\mathbf{k}^f\mathbf{A} + U^s(\mathbf{X}) + \frac{1}{2}\mathbf{\Phi}(\mathbf{X})\mathbf{k}^f\mathbf{\Phi}(\mathbf{X}) + \mathcal{Re}[\mathbf{\Phi}(\mathbf{X})\mathbf{k}^f\mathbf{A}]. \tag{4}$$

For both energies, the first term is the contribution of acoustics alone, the second and third ones are the contribution of the structure with its added fluid, i.e., an added mass for the kinetic energy and an added stiffness for the elastic energy, and the fourth term couples the acoustics with the structure. The presence of the added fluid field $\mathbf{\Phi}(\mathbf{X})$ in the coupling terms is not trivial, and constitutes a major feature of the fluid–structure interaction. As several fields $\mathbf{\Phi}$ can be chosen, several coupling terms can be obtained, and the question of the best choice of $\mathbf{\Phi}$ arises; this issue is partly addressed in Section 4. For the time being, it is enough to mention that the coupled modes obtained at the end of the calculation do not depend on the choice of the added field; different added fields would provide the same physical result, which is the thing that really matters.

The important point is that the fields $\mathbf{A}$ and $\mathbf{X}$ are independent, which was not the case of the former fields $\mathbf{X}^f$ and $\mathbf{X}$. The energies of Eqs. (3) and (4) can hence be used for determining the eigenmodes of the coupled system. As the goal of

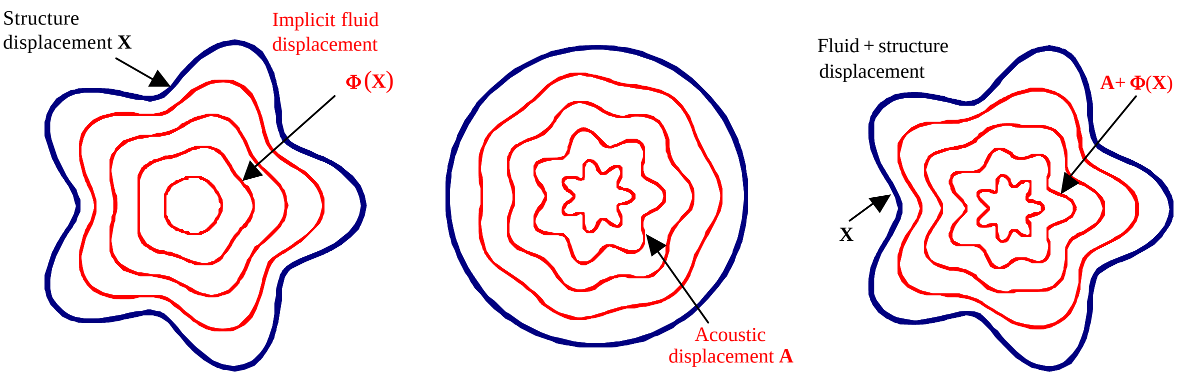

Fig. 1. Structural, acoustical and coupled modes of a cylinder with inner fluid.

the study is the determination of coupled modes from uncoupled modes, the direct resolution of the energy equations (3) and (4) is not investigated here. Let uncoupled modes be considered instead, and let the $\boldsymbol{\xi}_n$ be the N nondimensional eigenmodes of the structure without fluid, and let the $\boldsymbol{\alpha}_p$ be the P nondimensional eigenmodes of the acoustic subsystem. The displacement fields $\mathbf{X}$ and $\mathbf{A}$ of the coupled system can be expanded in the natural mode bases of the uncoupled subsystems

$$\mathbf{X} = \sum_n X_n \boldsymbol{\xi}_n \quad \text{and} \quad \mathbf{A} = \sum_p A_p \boldsymbol{\alpha}_p.$$

The above expansion of the displacement fields brings finally out the expressions of the mass and of the stiffness matrices

$$\mathbf{M} = \begin{pmatrix} m_n\delta_{nn'} + \boldsymbol{\Phi}(\boldsymbol{\xi}_n)\mathbf{m}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_{n'}) & \boldsymbol{\alpha}_p\mathbf{m}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_n) \\ \boldsymbol{\Phi}(\boldsymbol{\xi}_n)\mathbf{m}^f\boldsymbol{\alpha}_p & \boldsymbol{\alpha}_p\mathbf{m}^f\boldsymbol{\alpha}_p \end{pmatrix} \quad \text{and} \quad \mathbf{K} = \begin{pmatrix} k_n\delta_{nn'} + \boldsymbol{\Phi}(\boldsymbol{\xi}_n)\mathbf{k}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_{n'}) & \boldsymbol{\alpha}_p\mathbf{k}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_n) \\ \boldsymbol{\Phi}(\boldsymbol{\xi}_n)\mathbf{k}^f\boldsymbol{\alpha}_p & \boldsymbol{\alpha}_p\mathbf{k}^f\boldsymbol{\alpha}_p \end{pmatrix}, \tag{5}$$

where $\delta_{nn'}$ is equal to 1 if n equals n' and to zero otherwise, $m_n\delta_{nn'}$ and $k_n\delta_{nn'}$ are respectively the $N\times N$ modal mass and stiffness matrices of the structure without fluid, $\boldsymbol{\alpha}_p\mathbf{m}^f\boldsymbol{\alpha}_p$ and $\boldsymbol{\alpha}_p\mathbf{k}^f\boldsymbol{\alpha}_p$ are respectively the $P\times P$ acoustic modal mass and stiffness matrices, $\boldsymbol{\Phi}(\boldsymbol{\xi}_n)\mathbf{m}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_{n'})$ and $\boldsymbol{\Phi}(\boldsymbol{\xi}_n)\mathbf{k}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_{n'})$ are respectively the $N\times N$ added mass and stiffness matrices, and $\boldsymbol{\alpha}_p\mathbf{m}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_n)$ and $\boldsymbol{\alpha}_p\mathbf{k}^f\boldsymbol{\Phi}(\boldsymbol{\xi}_n)$ are respectively the $N\times P$ mass and stiffness coupling matrices. Incidentally, it can be shown by a counting of the degrees of freedom that the total number of coupled modes is simply $N+P$.

Eq. (5) provides the mass and stiffness matrices of a fluid–structure system in the uncoupled modes basis, with the help of the added fluid field. By construction, the matrices are symmetric, a point of major interest for computational efficiency: a large amount of research was made in the past decades to derive symmetric formulations of the FSI such as the "$u\ p\ \Phi$" method proposed by Morand and Ohayon (1992). The coupled modes of the system can be computed by determining the eigenmodes of the matrices of expression (5). From a physical point of view, the presence of nondiagonal coupling terms is consistent with the classical case of a pair of coupled oscillators (Rocard, 1971).

It is worth noting here that the structure could be described as well incorporating the added masses and the added stiffnesses, as for instance in seismic analysis of water piping systems, where the water mass is added to the mass of the structure. In such a case, the added masses and stiffnesses would vanish because they would already be incorporated in the structural modal masses and stiffnesses. The choice of the representation is a matter of convenience, depending on the type of calculation and the features of the computer codes available.

3. Energy balance of coupled modes

The issue is now to determine a relationship between the amplitudes of the acoustic displacement and of the structure. The theoretical properties of the natural modes can be used for that purpose. Let the Rayleigh quotient Q of the fluid–structure system be written as

$$Q = \frac{(k+k^{\mathrm{add}})X^2 + k^aA^2 + 2k^cAX}{(m+m^{\mathrm{add}})X^2 + m^aA^2 + 2m^cAX}, \tag{6}$$

where k and m stand respectively for the structure stiffness and mass, $k^{\mathrm{add}}X^2$ and $m^{\mathrm{add}}X^2$ stand respectively for the added stiffness term $\boldsymbol{\Phi}(\mathbf{X})\mathbf{k}^f\boldsymbol{\Phi}(\mathbf{X})$ and the added mass term $\boldsymbol{\Phi}(\mathbf{X})\mathbf{m}^f\boldsymbol{\Phi}(\mathbf{X})$, k^aA^2 and m^aA^2 stand respectively for the acoustic stiffness term $\mathbf{A}\mathbf{k}^f\mathbf{A}$ and the acoustic mass term $\mathbf{A}\mathbf{m}^f\mathbf{A}$, k^cAX and m^cAX stand respectively for the stiffness coupling term $\mathcal{R}e[\boldsymbol{\Phi}(\mathbf{X})\mathbf{k}^f\mathbf{A}]$ and the mass coupling term $\mathcal{R}e[\boldsymbol{\Phi}(\mathbf{X})\mathbf{m}^f\mathbf{A}]$.

It is well-known [see for instance Meirovitch (1967)] that the Rayleigh quotient is stationary for eigenmodes, and that the value of the quotient is then equal to the square of the circular natural frequency. This implies that if $\mathbf{A}$ and $\mathbf{X}$ describe an eigenmode, a small variation of either the amplitude of $\mathbf{A}$ or the amplitude of $\mathbf{X}$ leaves quotient (6) unchanged. Substituting $(1+\varepsilon)\mathbf{A}$ for $\mathbf{A}$ and $(1+\varepsilon')\mathbf{X}$ for $\mathbf{X}$ in expression (6), the variation of Q is at first order

$$\delta Q = \frac{2\varepsilon'(k+k^{\mathrm{add}})X^2 + 2\varepsilon k^aA^2 + 2(\varepsilon+\varepsilon')k^cAX}{(m+m^{\mathrm{add}})X^2 + m^aA^2 + 2m^cAX} - Q\frac{2\varepsilon'(m+m^{\mathrm{add}})X^2 + 2\varepsilon m^aA^2 + 2(\varepsilon+\varepsilon')m^cAX}{(m+m^{\mathrm{add}})X^2 + m^aA^2 + 2m^cAX}.$$

Replacing Q by the square of the natural frequency ω_e^2, and requiring the variation of Q to vanish at first order in ε and ε', one gets

$$(m^a\omega_e^2 - K^a)A^2 = [(m+m^{\mathrm{add}})\omega_e^2 - (k+k^{\mathrm{add}})X^2 = (m^a\omega_e^2 - k^a)AX. \tag{7}$$

Eq. (7) shows that the distribution of the kinetic and elastic energies of a coupled natural mode is such that the acoustical term, the structural term with the added fluid and the interaction term are balanced. It puts on equal footing the structure field with its added fluid and the acoustic field.

If the system were truly uncoupled, the interaction term of Eq. (7) would vanish. As a consequence, the acoustic elastic energy would be equal to the acoustic kinetic energy, which means that the total acoustic energy would be constant with time, and the same holds for the structure with its added fluid. Oppositely, if the interaction term does not vanish, the acoustic energy and the structure energy are time dependent, which means that energy is exchanged between the structure and the acoustic subsystem. The common value of the terms of Eq. (7) can then be said to measure the interaction energy between the degrees of freedom **A** and **X**.

4. Discussion

The kinematic approach described in Section 2 provides an explicit expression of the mass and of the stiffness matrix of a coupled system. The cornerstone of the approach is the concept of added fluid, and the question which arises is whether the added fluid is a genuine physical variable or a mere calculation step. As already mentioned, different added fluid fields can be defined for a coupled system, and there is no obvious way to define a ''best'' added fluid field for any possible situation. This should not be considered as a theoretical flaw, because the role of the added fluid field is to make calculations of coupled modes possible. Of course, one expects the final results of the calculations not to depend on the choice of the added fluid field. Special care must then be taken to discriminate between the variables that depend on the choice of the added fluid field, and the ones which do not. Keeping this distinction in mind, the general features of FSI can be discussed.

The discussion is split into three sections. In Section 4.1, the energy exchange and the fluid-to-structure amplitude are determined without the help of the kinematic approach, so that the general features of the coupling are highlighted in a neutral way. In Section 4.2, the concept of Φ-coupling is introduced, which stands for the mathematical interaction of the uncoupled modes of a coupled system. In Section 4.3, the physical coupling is reinvestigated with the help of the kinematic approach.

4.1. Physical definition of the coupling strength

In the present section, the kinematic approach is deliberately discarded, so that no question about the Φ-dependency of the coupling terms arises. As regards physics, the definition of the coupling strength may appear straightforward: one simply states that a system is strongly coupled if the fluid part and the structure part exchange a large amount of energy during one cycle. For an eigenmode described by the fluid displacement $\mathbf{X}_f$ and the structural displacement $\mathbf{X}$, a criterion can be elaborated, based on the fact that the total kinetic energy balances the total elastic energy. An interaction energy U^{int} can then be defined by

$$U^{\text{int}} = K^f(\mathbf{X}^f) - U^f(\mathbf{X}^f) \quad \text{and} \quad U^{\text{int}} = U^s(\mathbf{X}) - K^s(\mathbf{X}).$$

According to the aforementioned criterion, the first coupling indicator should compare the interaction energy U^{int} to some reference energy. It makes sense to use a global energy of the system, for instance the sum of the kinetic energies of the fluid and of the structure. An absolute coupling indicator is then defined by

$$\Theta = \frac{U^{\text{int}}}{K^f(\mathbf{X}^f) + K^s(\mathbf{X})}. \tag{8}$$

If the indicator Θ is close to zero, U^{int} must be small compared to the larger kinetic energy, which implies that the corresponding elastic energy must have about the same value. No more can be said about the two energies. In contrast, if the indicator Θ is not small (typically higher than 0.1), the total energy of the fluid and the total energy of the structure would be of the same order, but it could occur that the fluid would behave essentially as a mass and the structure as a stiffness for instance. The coupling indicator is hence not sufficient to describe completely the features of FSI.

Let now the response of a coupled mode under the effect of an harmonic fluid force $\mathbf{F}_\omega$ be investigated. From now on, the fluid and the structure displacements are equal to the modal displacements multiplied by an overall harmonic factor. As mentioned before, the difference between the elastic energy and the kinetic energy in the harmonic regime is the amplitude of the energy variation of the system during one cycle. This energy variation must be balanced by the work of the external force $\mathbf{F}_\omega$, and one easily gets

$$U^f(\mathbf{X}^f) + U^s(\mathbf{X}^s) - K^f(\mathbf{X}^f) - K^s(\mathbf{X}^s) = \mathbf{F}^*_\omega \cdot \mathbf{X}^f.$$

One would like to express the structure displacement as a force spectrum divided by a mass and by the difference $\omega^2 - \omega_e^2$, ω_e being the natural frequency of the pipe. This can be achieved by denoting by m the modal mass of the structure, m^f the modal mass of the fluid, $\omega_f^2 = \omega^2 U^f(\mathbf{X}^f)/K^f(\mathbf{X}^f)$ the Rayleigh frequency of the fluid and $\omega_s^2 = \omega^2 U^s(\mathbf{X})/K^s(\mathbf{X})$ the Rayleigh frequency of the structure

$$m^f(\omega_f^2 - \omega^2)X^{f2} + m(\omega_s^2 - \omega^2)X^2 = \mathbf{F}^*_\omega \cdot \mathbf{X}^f.$$

The left-hand terms can be rearranged by introducing the natural frequency, obtained by requiring that the kinetic energy of an eigenmode be equal to its elastic energy

$$(m^f X^{f2} + mX^2)(\omega_e^2 - \omega^2)X^2 = \mathbf{F}^*_\omega \cdot \mathbf{X}^f.$$

Let F_ω be equal to $\mathbf{F}^*_\omega \cdot \mathbf{X}^f$ divided by the amplitude X^f, and let the kinetic energies of the fluid and of the structure be reintroduced instead of the mX^2 and m^fX^f terms, the structural displacement can be expressed as the response of a single degree of freedom oscillator under the effect of an external force, the mass of the system being the square root of the product of the structure modal mass and the fluid modal mass

$$X = \frac{1}{\sqrt{K^f/K^s} + \sqrt{K^s/K^f}} \frac{1}{\sqrt{m^f m}} \frac{F_\omega}{\omega_e^2 - \omega^2}. \tag{9}$$

Eq. (9) is the theoretical response of a coupled mode to a fluid excitation $\mathbf{F}_\omega$ expressed in the ($\mathbf{X}_f$, $\mathbf{X}$) representation. The nondimensional factor of the right term describes the transfer of energy from the fluid to the structure, and its variations are plotted in Fig. 2.

The nondimensional energy transfer term constitutes a second indicator for the coupling intensity, different from the former one. As mentioned before, a low Θ indicator can be associated with either a low or a high energy transfer, and the same holds for a high Θ indicator. Further investigation is then required to better understand the features of FSI, and the kinematic approach is used for that purpose in the next sections.

4.2. The Φ-coupling concept

In Section 4.1, the strength of the coupling was defined in a physical way, and the kinematic approach was deliberately not used. The kinematic method leads to a mathematical concept of coupling, denoted as "Φ-coupling" in the following, which does not coincide with the physical coupling concept of Section 4.1, as will be shown in Section 4.3.

The Φ-coupling concept is based on a semi-arbitrary decomposition of the system in an acoustic part and in a structure part with added fluid. In the present section, the added fluid field $\mathbf{\Phi}$ is assumed to be a choice of modelling, and the physical interpretation of the Φ-coupling is not detailed. The issue is to determine coupling indicators relative to a given set of acoustic modes and structural modes. As a starting point, one has to compare the coupling terms of Eq. (4) to other energies. Obviously, no energy can be extracted from a structure at rest, so that the kinetic and elastic

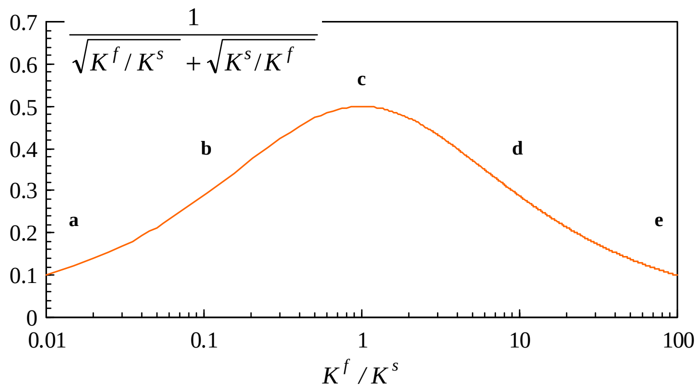

Fig. 2. Nondimensional energy transfer as a function of the kinetic energy ratio.

operators must be positive definite. The coupling terms can hence be considered as inner products of the fields $\mathbf{\Phi(X)}$ and $\mathbf{A}$, and coupling indicators can be proposed based on the comparison of the cross-product with the self-products of the fields, defining $\mathscr{C}_m[\mathbf{A}, \mathbf{\Phi(X)}]$ as the nondimensional mass-weighted spatial correlation of the displacement fields $\mathbf{A}$ and $\mathbf{\Phi(X)}$, and $\mathscr{C}_k[\mathbf{A}, \mathbf{\Phi(X)}]$ as their stiffness-weighted spatial correlation

$$\mathscr{C}_m[\mathbf{A}, \mathbf{\Phi(X)}] = \frac{\mathbf{\Phi(X)m}^f\mathbf{A}}{\sqrt{\mathbf{\Phi(X)m}^f\mathbf{\Phi(X)}}\sqrt{\mathbf{Am}^f\mathbf{A}}} \quad \text{and} \quad \mathscr{C}_k[\mathbf{A}, \mathbf{\Phi(X)}] = \frac{\mathbf{\Phi(X)k}^f\mathbf{A}}{\sqrt{\mathbf{\Phi(X)k}^f\mathbf{\Phi(X)}}\sqrt{\mathbf{Ak}^f\mathbf{A}}},$$

the equality being achieved if the fields $\mathbf{A}$ and $\mathbf{\Phi(X)}$ have the same shape. In other words, the coupling terms incorporate acceptance integrals, very similar to the ones encountered in flow-induced vibrations, where the spatial matching of the turbulence scales and of the structure modes is a key issue (Weaver et al., 2000; Au-Yang, 2001). Intuitively, the strength of the Φ-coupling should be related to the value of the correlation coefficients $\mathscr{C}_m$ and $\mathscr{C}_k$. The spatial matching of the modes is however not sufficient to define the intensity of the Φ-coupling. More convenient indicators can be obtained rewriting the balance equation (7) as

$$m^a(\omega_e^2 - \omega_a^2)A^2 = (m + m^{\text{add}})(\omega_e^2 - \Omega^2)X^2 = -(\eta^m\omega_e^2 - \eta^k\Omega\omega_a)AX\sqrt{m^a(m + m^{\text{add}})}. \tag{10}$$

where $\omega_a^2 = k^a/m^a$ is the Rayleigh quotient for the acoustic field, $\Omega^2 = (k + k^{\text{add}})/(m + m^{\text{add}})$ is the Rayleigh quotient for the structure with its added fluid field, and where two nondimensional coefficients have been introduced

$$\eta^m = \mathscr{C}^m[\mathbf{A}, \mathbf{\Phi(X)}]\sqrt{\frac{m^{\text{add}}}{m + m^{\text{add}}}} \quad \text{and} \quad \eta^k = \mathscr{C}^k[\mathbf{A}, \mathbf{\Phi(X)}]\sqrt{\frac{k^{\text{add}}}{k + k^{\text{add}}}}. \tag{11}$$

By construction, these nondimensional terms have a modulus lower than unity. What is more, they can have a modulus close to unity only (i) if the added fluid field and the acoustic field match and (ii) if the added mass/stiffness has a value of the same order as the structure mass/stiffness. In the other cases, η^m and η^k are necessarily small.

The coefficients η^m and η^k can be used as guides to define the Φ-coupling strength. First, the case where both indicators are small is considered. If the uncoupled frequencies ω_a and Ω differ, according to Eq. (10), either the acoustic component dominates and the coupled frequency is close to ω_a, or the structural term dominates and the coupled natural frequency is close to Ω. Hence, the coupled mode can be held as almost acoustic or almost structural.

One is thus led to the conclusion that a weakly Φ-coupled mode coincides, for its main component, with one of the uncoupled modes of the system, if the uncoupled frequencies do not coincide.

Of special interest is the case where the uncoupled natural frequencies ω_a and Ω coincide, and where η^m and η^k are small. Eq. (10) shows then that the kinetic energy of the structure with its added fluid equals the kinetic energy of the acoustics, and the mode cannot be held as almost acoustic nor almost structural. The question which arises is whether the acoustic part of the coupled mode is close to a purely acoustic mode, and whether the structure part of the mode is close to a structure with added fluid mode. There does not seem to exist a general answer, except if the coupling indicators are small for all the modes, in which case the coupled mode should really be the juxtaposition of an acoustic mode and of a structure with added fluid mode.

Finally, if η^m and η^k are not small (typically larger than 0.2), the components of the coupled mode may differ from uncoupled modes. This is however not a general statement, because a strongly coupled system with only one acoustic mode and one structural mode would obviously exhibit components similar to uncoupled modes [see the example in Moussou et al. (2000)].

To summarize, the best definition of the strength of the Φ-coupling should be global. If all modes are weakly coupled, the system should be considered as weakly Φ-coupled, and if some modes are strongly coupled, the system should be considered as strongly Φ-coupled, and a fully coupled analysis should be performed. Denoting by m_n the structure modal masses as in Section 2, the detection of a potential strong Φ-coupling can be made evaluating for all the uncoupled modes $\mathbf{A}_p$ and $\mathbf{X}_n$ the following indicators:

$$\eta_{np}^m = \frac{\mathbf{\Phi(X}_n)\mathbf{m}^f\mathbf{A}_p}{\sqrt{m_n + \mathbf{\Phi(X}_n)\mathbf{m}^f\mathbf{\Phi(X}_n)}\sqrt{\mathbf{A}_p\mathbf{m}^f\mathbf{A}_p}} \quad \text{and} \quad \eta_{np}^k = \frac{\mathbf{\Phi(X}_n)\mathbf{k}^f\mathbf{A}_p}{\sqrt{k_n + \mathbf{\Phi(X}_n)\mathbf{k}^f\mathbf{\Phi(X}_n)}\sqrt{\mathbf{A}_p\mathbf{k}^f\mathbf{A}_p}}. \tag{12}$$

4.3. *Physical coupling and Φ-coupling*

A strong physical coupling does not necessarily imply a strong Φ-coupling. As an illustration of this discrepancy, the case of a water piping system with the fluid described by plane waves and the structure by beams can be considered. Let the fluid field be expanded as the sum of an acoustic term, i.e., a field which vanishes at the wall, and of an added fluid field Φ, defined as the displacement of the neutral axis of the structure. It can easily be seen that this choice suits the needs of the continuity equation (1), and that the added fluid field deals mainly with the lateral displacement of the fluid, whereas the acoustic field deals with the relative axial displacement of the fluid. The trick is that a large amount of energy can be exchanged between the fluid and the structure by the lateral displacement, and in the same time that a small amount of energy can be exchanged by the axial displacement (typically in elbows and reducers). According to the physical criterion, the system is strongly coupled, but according to the kinematic expansion, a small amount of energy is exchanged between the acoustics and the rest of the system, so that the system is weakly Φ-coupled; the physical coupling and the Φ-coupling do not overlap.

Keeping this distinction in mind, the physical coupling can be investigated in more details with the help of the kinematic approach. As a first step, the Φ-coupling was defined as strong if the coefficient η^m or the coefficient η^k is close to unity, and as weak if both coefficients are small. Let the balance equation (10) of Section 4.2 be considered: in a manner similar to Section 4.1, the Φ-coupling can be defined as weak if the coupled frequency ω_e is close to Ω or close to ω_a.

As a second step, let the coupling indicator Θ defined by Eq. (8) be determined. Expanding the fluid field $\mathbf{X}^f$ and using the notations of Section 4.2, one gets

$$\Theta = \frac{(\omega_e^2 - \omega_s^2) m X^2}{\omega_e^2 \left(m^a A^2 + m X^2 + m^{\text{add}} X^2 + 2\eta^m A X \sqrt{m^a (m + m^{\text{add}})} \right)}.$$

Using the balance equation (10), one gets

$$\Theta = \frac{\mu - \kappa \dfrac{\Omega^2}{\omega_e^2}}{1 + 2\eta^m \sqrt{\dfrac{\omega_e^2 - \Omega^2}{\omega_e^2 - \omega_a^2}} + \dfrac{\omega_e^2 - \Omega^2}{\omega_e^2 - \omega_a^2}}, \tag{13}$$

where μ is the mass ratio $m/(m + m^{\text{add}})$ and κ is the stiffness ratio $k/(k + k^{\text{add}})$.

One can remark that if the natural frequency of the coupled system is close to ω_a, the denominator in Eq. (13) is large and the interaction energy is weak, but if the natural frequency of the coupled system is close to Ω, the interaction energy is not small, because the coupling indicator becomes equal to μ–κ; the added mass and the added stiffness generate an exchange of energy between the fluid and the structure, as was mentioned in the introduction of the section.

Two distinct situations arise according to the values of the added mass and stiffness. First, if both added mass and stiffness are small, the coupling indicators η^m and η^k are small as well by definition [see expression (11)], and the coupled natural frequency is either close to ω_a or to Ω according to the balance equation (10). As a consequence, the coupling indicator Θ is small as well, as Eq. (13) shows. One comes then to the conclusion that if its added mass and its added stiffness are small, a system is weakly coupled. This result generalizes a conclusion derived by Moussou et al. (2000), according to which a gas system would always be weakly physically coupled. As already mentioned, this does not imply that the amplitude of its vibrations should be small.

Second, if either the added mass or the added stiffness is not small, two sub-cases can occur. If the coupling indicators η^m and η^k are small because the shapes of the acoustic mode and of the structural mode do not match, the coupled frequency is either close to the structure frequency Ω, and the energy transfer is large because of the added mass and the added stiffness effect, or the coupled frequency is close to ω_a, and the energy transfer is small. Finally, if the coupling indicators η^m and η^k are not small because the shapes of the acoustic and of the structural modes match, the coupled frequency ω_e is not close to ω_a neither to Ω, and in most practical cases, the coupling indicator given by Eq. (13) is not small.

A strongly coupled system would then either be strongly Φ-coupled, or exhibit a large added mass or a large added stiffness. The other way around, one would expect a system which is weakly coupled to be weakly Φ-coupled. Unfortunately, there is no mathematical way to prove it, and a bad choice of the added fluid field may in some cases lead to a strong Φ-coupling, whereas the system under study would actually be weakly coupled. A practical way to avoid that situation would be to determine the indicator Θ afterwards, using the kinematic approach; if this indicator appears to be small while the Φ-coupling is strong, one can suspect a bad choice of the added field Φ. Furthermore, if the coupled indicators η^m and η^k have a value close to -1, one should suspect a bad choice of the added fluid field as well, because the added field Φ and the acoustic field $\mathbf{A}$ act in opposite directions.

Let now the kinetic energy ratio of Eq. (9) be expressed as

$$\frac{K^f}{K^s} = \frac{1}{\mu} \frac{m^a A^2 + m^{\text{add}} X^2 + 2\eta_m AX\sqrt{m^a(m + m^{\text{add}})}}{(m + m^{\text{add}})X^2},$$

which, using the balance equation (10), yields

$$\frac{K^f}{K^s} = \frac{1}{\mu}\left(\frac{\omega_e^2 - \Omega^2}{\omega_e^2 - \omega_a^2} + \frac{m^{\text{add}}}{m + m^{\text{add}}} + 2\eta^m\sqrt{\frac{\omega_e^2 - \Omega^2}{\omega_e^2 - \omega_a^2}}\right). \tag{14}$$

The discussion about the transfer function amplitude initiated in Section 4.1 can now be successfully completed using expression (14). First, let the case of a weakly coupled system be considered, with a choice of the added field which makes it weakly Φ-coupled. As established before, the added mass must be small compared to the structure mass, and the transfer function term is almost equal to the first term of Eq. (14). Hence, if the acoustic frequency differs from the structure frequency, the transfer function term is either very small or very large, and the nondimensional factor of Eq. (9) is small; only a small amount of energy is transferred from the fluid to the structure (case "a" or "e" of Fig. 2). Oppositely, if the acoustic frequency is equal to the structure frequency, the transfer function term is almost equal to unity, and the nondimensional factor of Eq. (9) has its maximum value; a large amount of energy is transferred from the fluid to the structure (case "c" of Fig. 2). What is more, the distribution of energy between the fluid and the structure does not depend on the value of the coupling indicators η_k and η_m if they are small enough. This could appear paradoxical at first sight because, in the extreme situation where the coupling terms are equal to zero, there cannot be any energy transfer from fluid to structure. The explanation can be found by replacing the asymptotic Fourier analysis by a time domain analysis. If the system is excited by a Dirac pulse, the harmonic regime described by the Fourier analysis is obtained after a time depending on the coupling terms; the lower the coupling terms, the longer the stabilization time.

Let the case of a strongly coupled system be now considered. Either the added mass is large or the coupled frequency is different from ω_a and ω_s, and the transfer function term of equation expression (14) is not equal to unity, but it cannot be very large nor very small. The nondimensional factor of Eq. (9) has a medium value, and a medium amount of energy is transferred from the fluid to the structure (case "b" or "d" of Fig. 2). What is more, the amplitude of the transfer function is not very sensitive to the difference between the acoustic frequency and the structure frequency; no "super-resonance" can occur due to fluid–structure interaction in a strongly coupled system.

Abrupt increases of the vibration level are then characteristic of weakly coupled systems, and not of strongly coupled systems. The practice of avoiding the coincidences of uncoupled frequencies in gas systems is sound (API 618, 1995), because a separation of the frequencies reduces the maximum value of the transfer function of the system.

To summarize, the amplitude of steady state vibrations determined by a coupled calculation is lower than the one which would be obtained by performing an acoustical calculation and applying the pressure as excitation forces to the structure. This suggests that a coupled calculation should generally be less penalizing that an uncoupled one, even if opposite results have sometimes be obtained in transient analysis (Wiggert and Tijsseling, 2001). Hence, the "no coincidence" recommendation of the design guides has a sound basis, because many industrial systems would exhibit a low-coupling behavior, due to the fact that the strong coupling conditions are not easily met.

5. Applications

Classical issues of FSI are now studied with the method derived in the present paper. Attention is focused on the understanding of the physical ideas, and not on the calculation by itself, so that the results are not detailed till the end.

5.1. Two coaxial cylinders with quiescent fluid

A classical case of fluid–structure interactions is the evaluation of the added mass for a pair of coaxial cylinders separated by a thin layer of quiescent fluid. Based on the resolution of the hydrodynamic pressure equation, analytical solutions are well-known (Païdoussis, 1998; Axisa, 2001; Gibert, 1988; Au-Yang, 2001), but physical features of the coupling can be highlighted with the help of the kinematic approach. The two cylinders are subjected to shell deformation, as shown in Fig. 3. In order to simplify things, the fluid is assumed incompressible and all displacements are assumed planar.

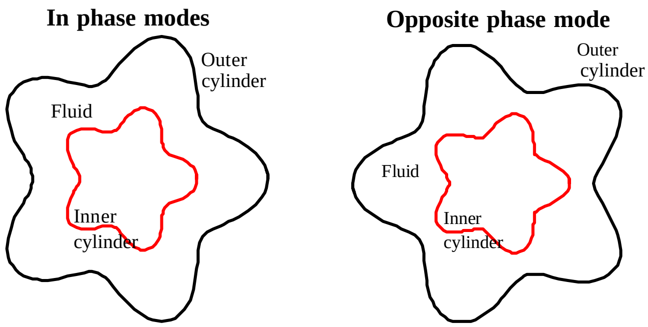

Fig. 3. Shell deformation of two coaxial cylinders coupled by quiescent fluid.

Let then the natural modes of each cylinder be described by the radial displacements $R_i \cos(i\theta + \varphi_i)$ and $r_j \cos(j\theta + \xi_j)$ at the outer and inner cylinders respectively. An added fluid field is now to be defined for the modes of each cylinder. The easiest way to get a convenient expression is based on the hydrodynamic equation of the fluid. This is mathematically similar to the pressure-based resolution in the literature, but the physical ideas are somehow different. In the hydrodynamic approach, the fluid velocity is proportional to the gradient of the pressure, and the pressure obeys the Laplace equation $\Delta p = 0$. Using combinations of terms

$$p_q = A_q r^q \cos(q\theta + \xi_q) + B_q r^{-q} \cos(-q\theta + \xi_q) \quad \text{for } q = i \quad \text{and} \quad q = j.$$

FSI-compliant fluid fields for the inner and the outer cylinder can be obtained in polar coordinates by application of the gradient operator. The boundary conditions demand the fluid displacement have on one side its normal component equal to the normal component of the cylinder displacement, and its normal component equal to zero on the other cylinder. The result is straightforward and is not detailed here; suffice it to mention that the fluid field is the sum of a $\mathbf{X}_i$ field proportional to the gradient of a pressure field p_i and compliant with the outer boundary condition on the one hand, and a $\mathbf{X}_j$ field proportional to the gradient of a pressure field p_j and compliant with the inner boundary condition on the other hand.

The expression of the kinetic energy of the system involves a cross-integration of the fluid fields $\mathbf{X}_i$ and $\mathbf{X}_j$. It can easily be seem by integrating upon cylinder lines that the cross-term is zero if the mode numbers i and j are not equal. As a consequence, the coupled modes of the two cylinders involve the same mode number for the inner and the outer cylinder. Furthermore, the Rayleigh quotient being stationary, the cross-term must be either minimum or maximum, which implies that the cylinder deformations are either in phase or in opposite phase, as shown in Fig. 3.

The calculation is not carried on any further, because the results would be identical to the ones in the literature.

5.2. The straight pipe with closed ends

Classical cases of coupled systems are given by straight pipes with closed ends described by 1-D compression waves (Tijsseling, 1996, 2002). As an illustration, the system with a clamped left end and a free right end is considered, as shown in Fig. 4.

Coupling effects occur due to the free closed end on the right side. The issue is to determine the coupled modes and the coupled natural frequencies by the kinematic approach.

The acoustic natural modes are $\boldsymbol{\alpha}_p = \sin p\pi x/L$ and the structural natural modes are $\boldsymbol{\xi}_n = \sin(n - 1/2)\pi x/L$, the first ones being plotted in Fig. 5. The added field is chosen as $\boldsymbol{\Phi}(\mathbf{X}) = \mathbf{X}$, so that $\boldsymbol{\Phi}(\boldsymbol{\xi}_n) = \boldsymbol{\xi}_n$. The mass matrix of the coupled system can then be determined as indicated in Section 2: the structure mass matrix is $m_{nn'} = (\rho_s S_s + \rho_f S_f)L \int \boldsymbol{\xi}_n \boldsymbol{\xi}_{n'}\, \mathrm{d}u$, the acoustic mass matrix is $m_{pp'} = \rho_f S_f L \int \boldsymbol{\alpha}_p \boldsymbol{\alpha}_{p'}\, \mathrm{d}u$, and the coupled mass matrix is $m^c_{np} = \rho_f S_f L \int \boldsymbol{\xi}_n \boldsymbol{\alpha}_p\, \mathrm{d}u$. The stiffness matrix is determined the same way: the structural stiffness matrix is $k_{nn'} = (ES_s + \rho_f c^2 S_f)/L \int \boldsymbol{\xi}'_n \boldsymbol{\xi}'_{n'}\, \mathrm{d}u$, the acoustic stiffness matrix is $k_{pp'} = \rho f c^2 S_f / L_f \int \boldsymbol{\alpha}_p \boldsymbol{\alpha}_{p'}\, \mathrm{d}u$, and the coupled stiffness matrix is $k^c_{np} = \rho_f c^2 S_f / L_f \int \boldsymbol{\xi}_n \boldsymbol{\alpha}_p\, \mathrm{d}u$.

A reference solution is given by the separate compression equations of the fluid and of the structure. As the displacements of both fluid and structure vanish on the left-hand side, only sine terms need to be considered

$$X = B \sin \omega x / c_s, \quad X_f = B_f \sin \omega x / c.$$

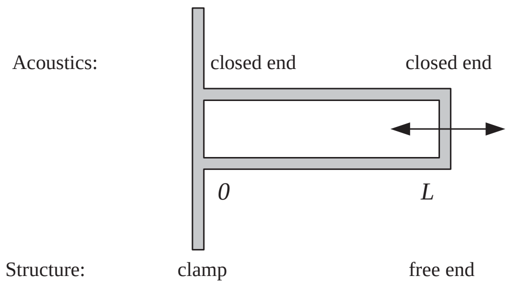

Fig. 4. A straight pipe with closed ends.

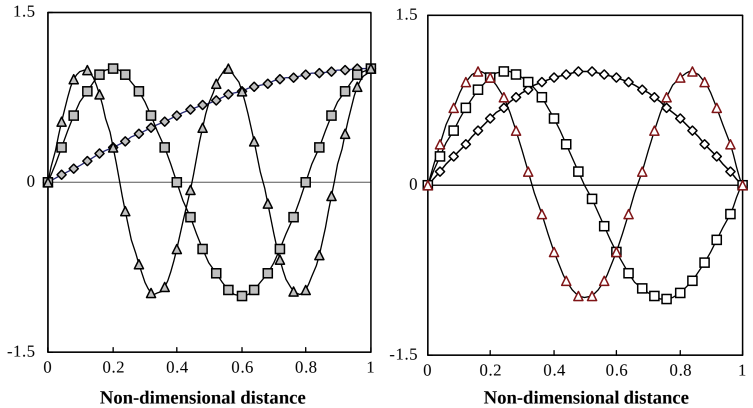

Fig. 5. Structural and acoustic uncoupled natural modes #1 (◇), #2 (□) and #3 (△) of the straight pipe.

On the right-hand side, the continuity of displacement at L results in

$$B_s \sin \omega L/c_s = B_f \sin \omega L/c,$$

and the dynamic balance at L brings out

$$B_s E S_s L/c_s \cos \omega L/c_s + B_f \rho_f c^2 S_f L/c \cos \omega L/c = 0.$$

Eliminating the amplitude terms B_s and B_f, one gets the dispersion equation

$$\frac{\rho_f c c_s S_f}{E S_s} \tan \frac{c}{c_s} \frac{\omega L}{c} + \tan \frac{\omega L}{c} = 0.$$

Dividing the abscissa x by the length L and defining a nondimensional frequency ω_{red} with the help of the mass $(\rho_s S_s + \rho_f S_f)L$ and of the stiffness $(E S_s + \rho_f c^2 S_f)/L$, the description of the system can be made nondimensional. The dispersion equation is then

$$\frac{\rho_f c c_s S_f}{E S_s} \tan \omega_{\text{red}} \frac{c}{c_s} \sqrt{\frac{1}{c^2} \frac{\rho_f c^2 S_f + E S_s}{\rho_f S_f + \rho_s S_s}} + \tan \omega_{\text{red}} \sqrt{\frac{1}{c^2} \frac{\rho_f c^2 S_f + E S_s}{\rho_f S_f + \rho_s S_s}} = 0.$$

Note that the term "dispersion equation" is here conventional, because the system considered is actually nondispersive as the speeds c and c_s do not depend on the frequency.

Let the fluid be water and the structure steel. The values of the densities, the speeds of sound and Young's modulus are hence determined (chosen here equal to 1000 and 7800 kg/m^3, 1500 m/s and 5063.7 m/s, 2×10^{11} Pa, respectively), so that the solutions of the dispersion equation depend only on the cross-section ratio S_f/S_s . Let this ratio be equal to 10 so that the masses of the fluid and of the structure are of the same order. As regards the exact solution, the dispersion equation can be numerically solved, and the amplitudes of the natural modes are easily determined by the continuity of displacement. At the same time, the approximate solution is derived using the kinematic approach based on the

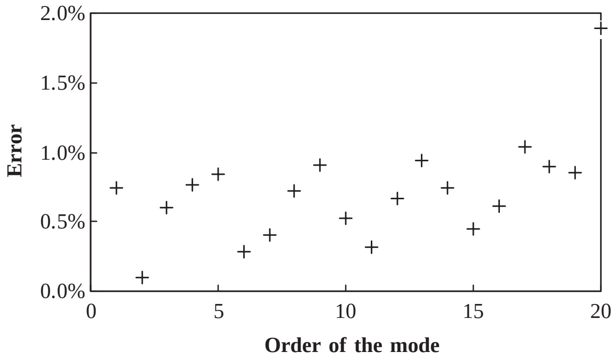

Fig. 6. Relative errors in the natural frequencies.

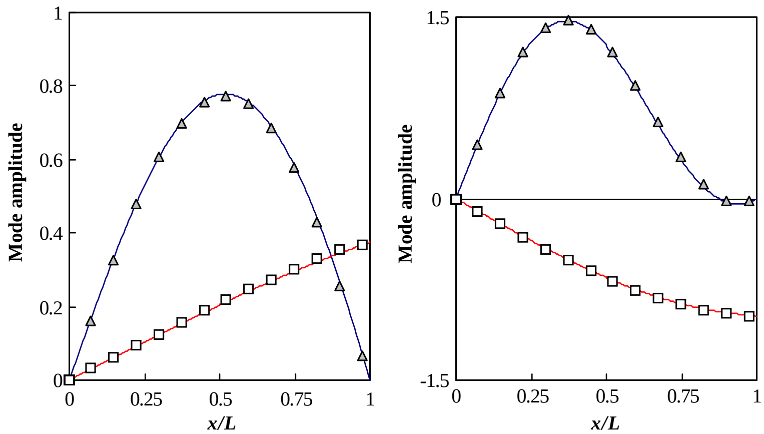

Fig. 7. Natural modes #1 and #2: △, acoustic displacement; □, structural displacement; –, exact values.

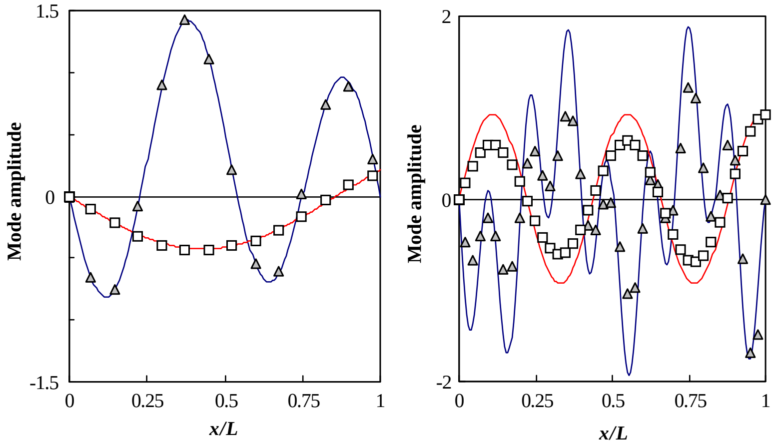

Fig. 8. Natural modes #5 and #20: △, acoustic displacement; □, structural displacement; –, exact values.

uncoupled modes of Fig. 5. Five structural modes and 15 acoustic modes were used to build up the coupled modes. The natural frequencies are estimated with an accuracy better than 1% for most cases as shown in Fig. 6.

The natural modes are well estimated too, as shown in Figs. 7 and 8. Even in the worst case (coupled mode #20 for 15 acoustic and 5 structural modes), the shape of the coupled mode is correctly obtained. Note that the structure parts of the modes are sine functions, whereas the acoustic parts of the modes are not, because they are equal to the differences of the fluid fields and of the added fluids, which are sine functions with different wavenumbers.

5.3. Low-frequency coupling in a Z-shaped pipe

Piping systems with elbows constitute simple structures prone to FSI effects. Let such a system be described by a Z-shaped pipe clamped at both ends as shown in Fig. 9.

The geometric parameters of the pipe are the following: length of each vertical part: 2.4 m, length of the horizontal part: 4.33 m, curvature radius of the elbows: 0.45 m, external diameter: 0.3 m, thickness: 0.005 m, Young's modulus: 2×10^{11} Pa, Poisson coefficient: 0.3, fluid density: 1000 kg/m^3, structure density: 7800 kg/m^3, elbow flexibility factor: 15.95, fluid sound velocity: 1500 m/s.

As a finite element analysis proved (Moussou et al., 2000), the first natural frequencies are below 20 Hz, and the first structural in-plane mode can be modelled in a simplified manner using the following hypotheses for the structure: (i) the vertical parts bend in the x direction, and behave as quasi-static beams, (ii) the elbows are flexible so that the moments at both ends of the horizontal part are equal to zero, and (iii) the horizontal part is a rigid inert body.

Focusing the attention on this first mode, the compressibility of the fluid can be neglected because the length of the pipe is much smaller than the acoustic wavelength of the fluid. Within this framework, the bare structure behaves as a single degree of freedom oscillator, with a mass equal to the one of the horizontal part, and a stiffness equal to the cantilever beam stiffness of the vertical parts. The issue is to determine the harmonic behavior of the pipe when a liquid fluid fills it. A complete analytical approach of this system can be found in Moussou et al. (2000), but the approach proposed here provides the essential results in a much simpler way. Let the added fluid field be simply equal to the axial displacement of each pipe. Neglecting the compression of the structure, it can easily be seen that the hydrodynamic velocity of the fluid is uniform along the pipe because of mass conservation, whereas the added fluid velocity is equal to zero in the bending pipes, and equal to the structure velocity in the central pipe. Hence, the kinetic and elastic energies of the pipe can be written in a straightforward way as

$$K=\underbrace{\frac{m_{\text{hor}}\omega^2x^2}{2}}_{\substack{\text{structure}\\\text{kinetic energy}}}+\underbrace{\frac{(2m^f_{\text{vert}}+m^f_{\text{hor}})\omega^2a^2}{2}}_{\text{fluid kinetic energy}}+\underbrace{m^f_{\text{hor}}\omega^2ax}_{\text{coupling term}}\quad\text{and}\quad U=\underbrace{\frac{kx^2}{2}}_{\text{structure elastic energy}},$$

where m_{hor} is the structure mass of the horizontal part of the pipe, k is the bending stiffness of the two vertical parts of the pipe, $m^f{}_{\text{hor}}$ is the fluid mass of the horizontal part of the pipe and $m^f{}_{\text{ver}}$ is the fluid mass of each vertical part of the pipe.

Whatever the acoustic boundary conditions, the coupling is described by the coupling mass m^f_{hor} only. For the sake of illustration, let the acoustic boundary conditions be zero pressure at each end. The nondimensional coupling coefficient of the Section 4.2 takes the value

$$\eta_m=\frac{m_c}{\sqrt{m^f m}}=\sqrt{\frac{\rho_f S_f}{\rho_s S_s+\rho_f S_f}}\,\frac{L_{\text{hor}}}{L_{\text{hor}}+2L_{\text{vert}}}\approx 0.38.$$

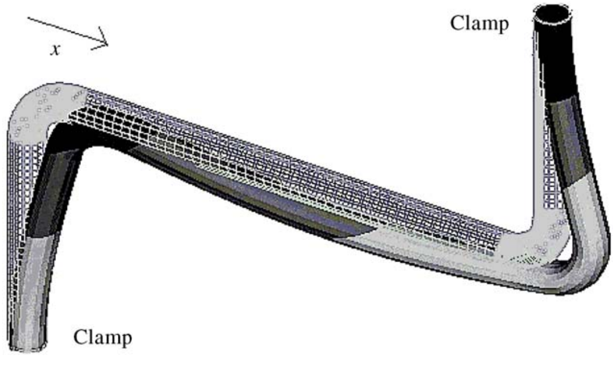

Fig. 9. A Z-shaped piping system and its first plane mode.

This value is very high, so that the coupling is strong in this case. It explains why no sharp increase of the transfer function was observed in the earlier paper (Moussou et al., 2000) when the acoustic and structural natural frequencies came to coincide. It suggests that strong coupling occurs in liquid-filled pipes preferentially in the low frequency range.

The method proposed in the present paper leads to the essential results in a simpler way than classical methods. As the other results are straightforward, they are not detailed her.

5.4. Dispersion relations and coupling

Dispersion relations are sometimes applied to systems with one dimension much larger than the acoustic and the structural wavelengths, such as long thin pipes submitted to shell deformation. Basically, the idea is to superpose the dispersion relations of the fluid and of the structure (Fahy, 1985; De Jong, 1994; Caillaud et al., 2003); the natural frequencies of the system are frequencies for which the wavenumbers of the structure and of the fluid are identical.

As an illustration, let a straight cylindrical pipe with thin walls be considered. Following Fahy (1985), the acoustic modes are combinations of elementary functions based on the following nondimensional pressures

$$p_{mn}(r,\theta,z) = \cos n\theta \mathrm{J}_n(k_r^{nm} r)\cos(k_z^{nm} z),$$

J_n being a Bessel function of order n, and the radial wavenumber κ_r being determined by the zero displacement condition at the wall: $\mathrm{J}'(\kappa_r^{nm} R) = 0$. Multiple zeros of the derivative of the Bessel function exist, so that an index m is introduced in the expression of the radial wavenumber, which corresponds to the number of circular pressure nodes.

As regards the structure, the shell deformation is found to be a combination of terms depending on one index n', the orthoradial displacement of the mean radius R having the following form

$$w_n(\theta, z) = \cos n'\theta \cos(\lambda_z^{n'} z).$$

This expression applies to a structure without fluid, but any added fluid with the following expression can be used to define added-fluid modes

$$\varphi_{n'} = f(r/R)\cos n'\theta \cos(\lambda_z^{n'} z) \quad \text{with } f(1) = 1,$$

because such added fluid associated with different indexes n' are orthogonal.

The kinematic approach can now be used to justify the dispersion relation approach. The comparison of the acoustic pressure p_{mn} and of the added fluid field $\varphi_{n'}$ proves that only modes with the same radial dependence (i.e. $n = n'$) can interact; otherwise, the interaction integral is equal to zero. For the same reason, the axial wavenumbers k_z^{nm} and $\lambda_{n'}$ must be equal too to ensure interaction of the modes. Hence, the coupled modes are described by one orthoradial index n, associated with one wavenumber κ_z^{nm} for the fluid and for the structure.

Two cases must now be considered. If the system is weakly coupled (for instance, if the fluid is steam), then the vibrations of the structure can become significant if the natural frequencies of the uncoupled systems are identical (see Section 4.3). Hence, the frequencies where vibrations are likely to occur are the ones for which

$$\kappa_z^{nm}(\omega) = \lambda_z^{n'}(\omega),$$

which is precisely the dispersion equation proposed by Fahy (1987). It is worth noting that the author of this last reference recommends the use of this formula in situations where "*the coupled modes resemble closely their uncoupled components*", a condition which is equivalent to the weak coupling of the present paper.

In contrast, if the system is strongly coupled, the dispersion relations of the uncoupled components cannot be used, and the natural modes of the coupled system should be looked for on the basis of a fully coupled calculation [see De Jong (1994) for instance].

6. Conclusion

A kinematic variational method for FSI in linear conservative systems was proposed, based on the concept of added fluid field, which stands for the implicit or explicit fluid displacement associated with a structural displacement in a purely mechanical approach. It is shown that the coupled natural modes can be deduced from the uncoupled modes by a procedure involving symmetric mass and stiffness matrices. Indicators of the strength of the coupling are provided: it is shown that a system can be strongly coupled if its added mass or its added stiffness are large, or if one of its acoustic modes matches one of its structural modes, and if the fluid mass/stiffness has a value of the same order as the structural mass/stiffness. Weakly coupled systems can be exposed to large variations of their vibration level if their uncoupled natural frequencies come to coincide, whereas the behavior of strongly coupled systems depends less on the uncoupled frequency values.

The kinematic approach was used to revisit case studies of fluid structure interactions. The coupling features of a pair of coaxial cylinders with a quiescent fluid are deduced in a straightforward manner. A straight pipe with closed ends was successfully used as a validation case for the method, and a calculation of the coupled mode of a Z-shaped pipe is derived in a much simpler way than in a former presentation. Comments on the dispersion relations of Fahy (1987) are provided for straight pipes submitted to shell deformation and nonplanar acoustic propagation, with the help of the kinematic method.

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